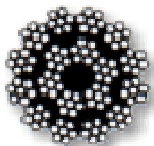
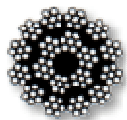
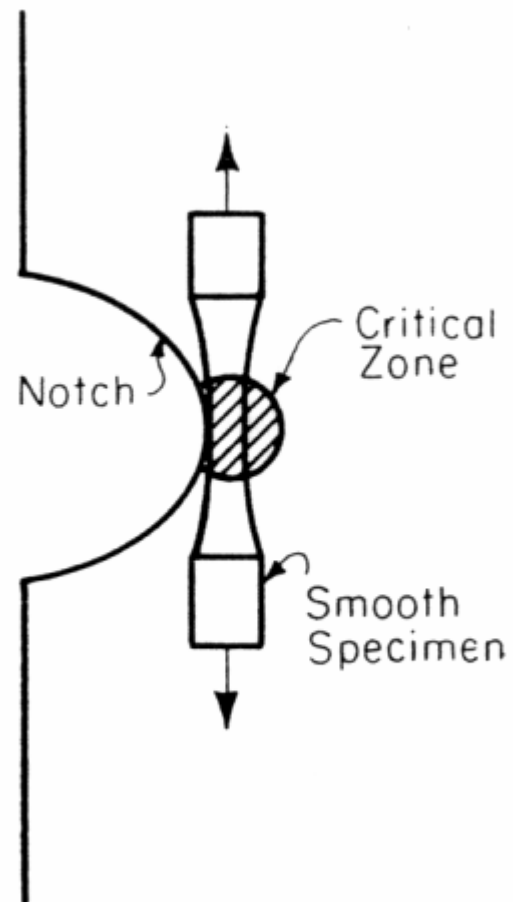


7. Strain-life přístup



Úvod



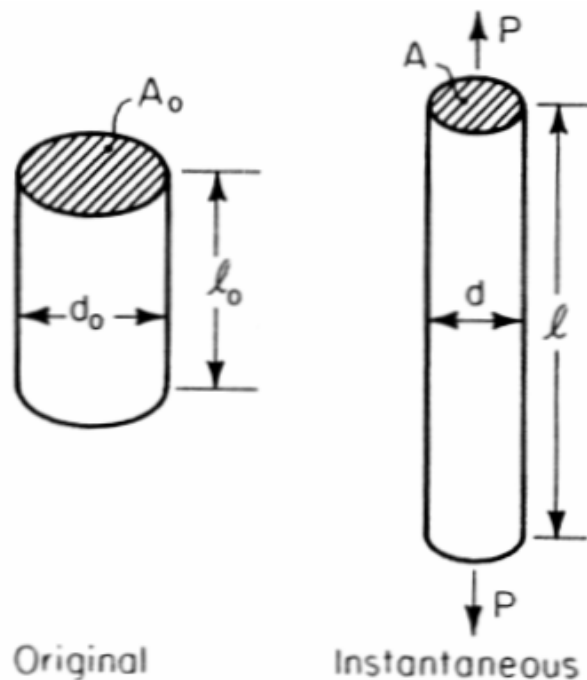
Úvod

$$S = \text{engineering stress} = \frac{P}{A_0}$$

-inženýrské napětí

$$e = \text{engineering strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

- inženýrské přetvoření



P = applied load

l_0 = original length

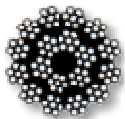
d_0 = original diameter

A_0 = original area

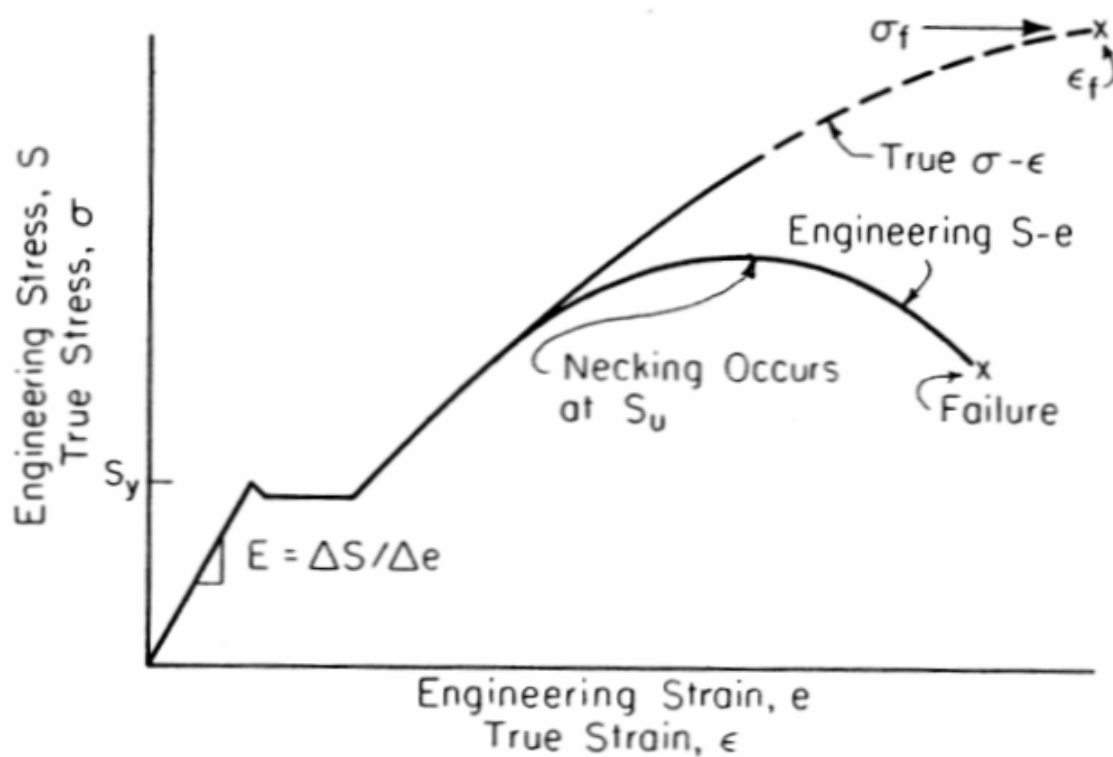
l = instantaneous length

d = instantaneous diameter

A = instantaneous area



Úvod



Porovnání křivek skutečného přetvoření versus skutečné napětí a inženýrské přetvoření versus inženýrské napětí

Skutečné napětí:

$$\sigma = \text{true stress} = \frac{P}{A}$$

Skutečné přetvoření:

$$\epsilon = \text{true strain} = \int_{l_0}^l \frac{dl}{l} = \ln \frac{l}{l_0}$$

l – délka po deformaci:

$$l = l_0 + \Delta l$$

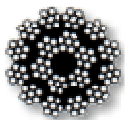
$$\epsilon = \ln \frac{l_0 + \Delta l}{l_0} = \ln \left(1 + \frac{\Delta l}{l_0} \right)$$

$$\epsilon = \ln(1 + e)$$

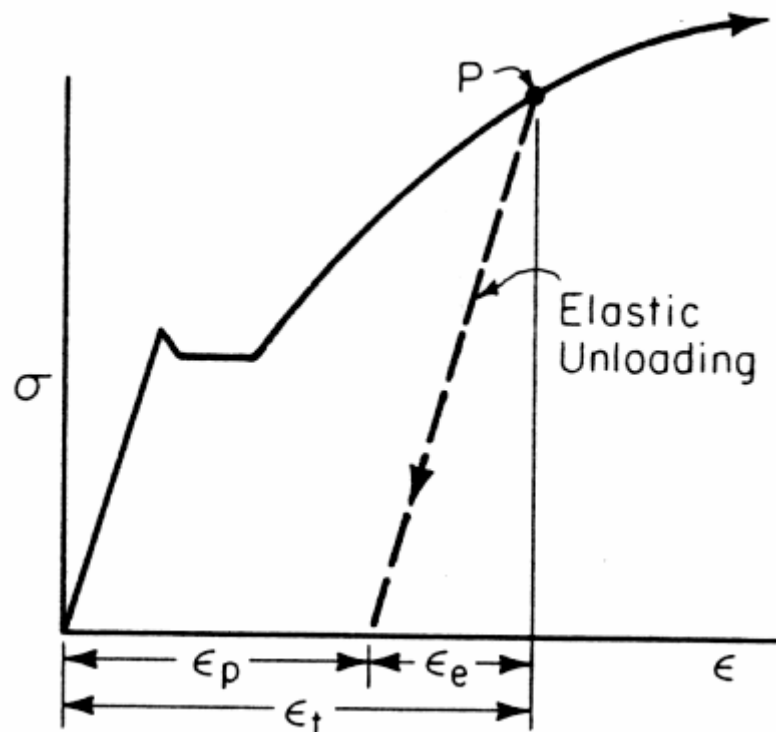
$$\epsilon = \ln(1 + e) = \ln \frac{A_0}{A}$$

$$\frac{A_0}{A} = 1 + e$$

$$\sigma = S(1 + e)$$



Úvod



Celkové přetvoření:

$$\epsilon_t = \epsilon_e + \epsilon_p$$

elastické přetvoření

plastické přetvoření

$$\sigma = K(\epsilon_p)^n \quad \epsilon_p = \left(\frac{\sigma}{K}\right)^{1/n}$$

n ... exponent zpevnění

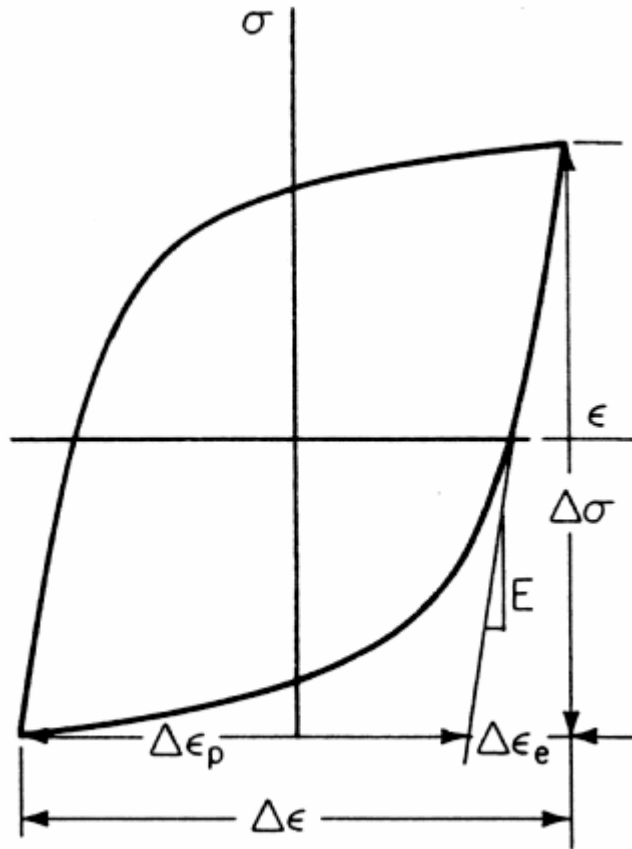
K ... koeficient zpevnění

$$\epsilon_e = \frac{\sigma}{E}$$

$$\epsilon_t = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$



Úvod

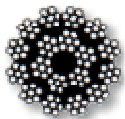


$$\epsilon_a = \frac{\Delta\epsilon}{2} \quad \sigma_a = \frac{\Delta\sigma}{2}$$

$$\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_p$$

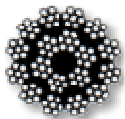
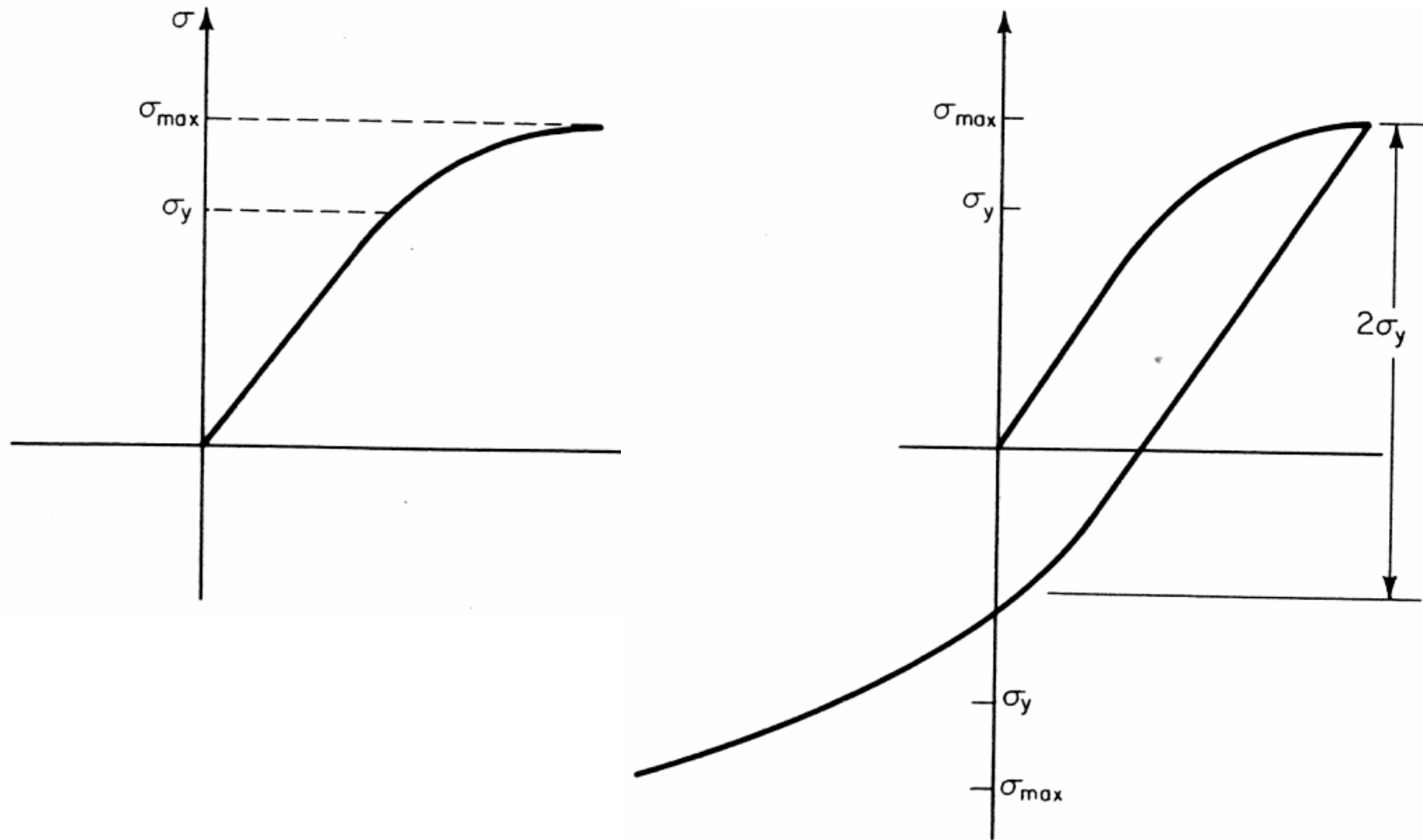
$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2}$$

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\epsilon_p}{2}$$



Úvod

Bauschingerův efekt



Úvod

Cyklické zpevnění

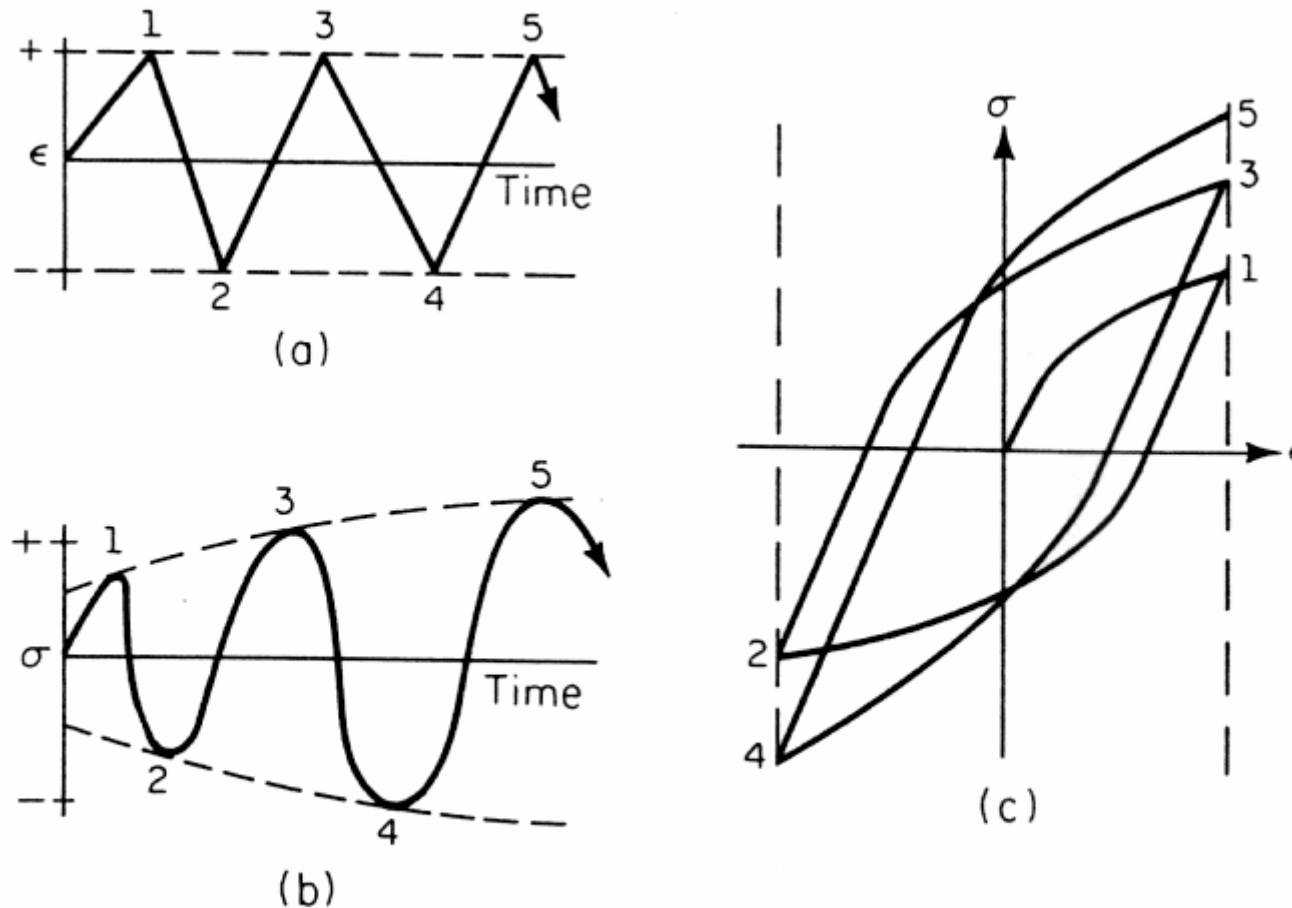
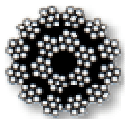


Figure 2.7 Cyclic hardening: (a) constant strain amplitude; (b) stress response (increasing stress level); (c) cyclic stress–strain response.



Úvod

Cyklické změkčení

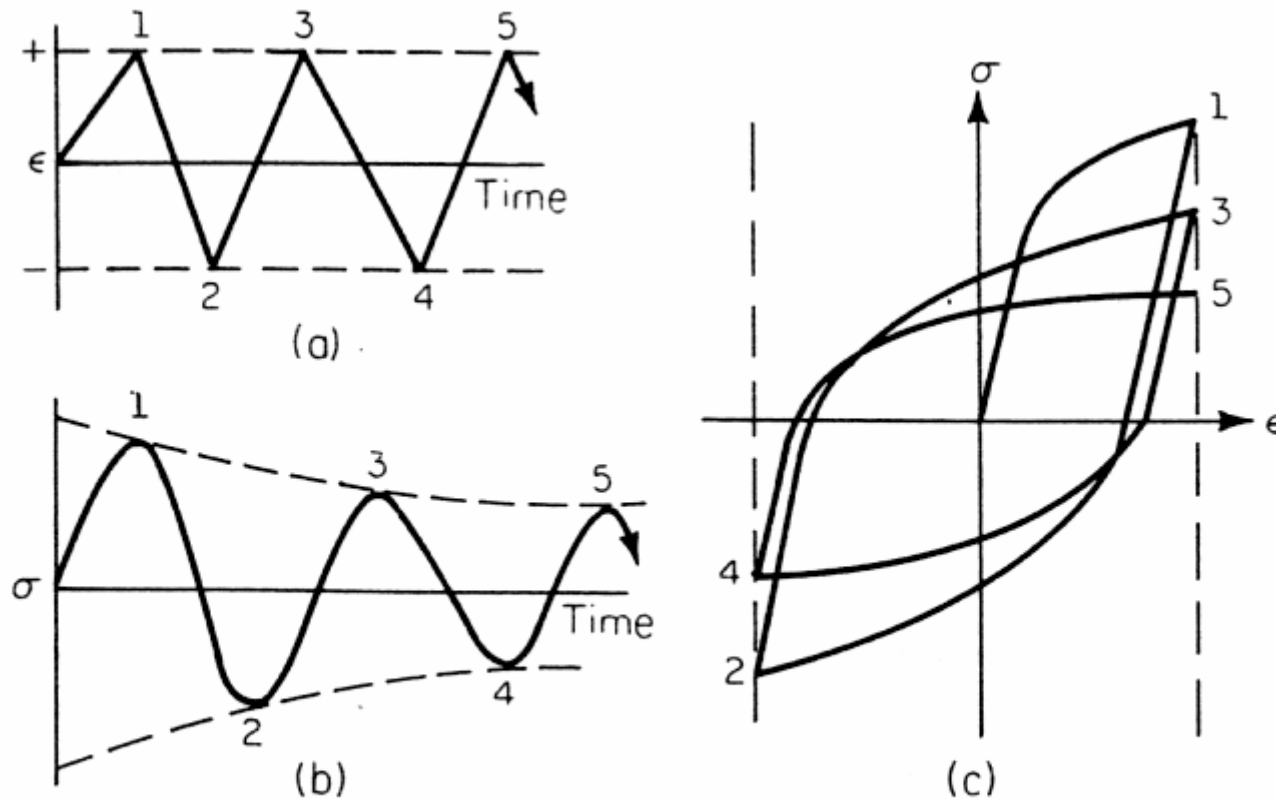
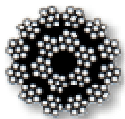


Figure 2.8 Cyclic softening: (a) constant strain amplitude; (b) stress response (decreasing stress level); (c) cyclic stress–strain response.



Úvod

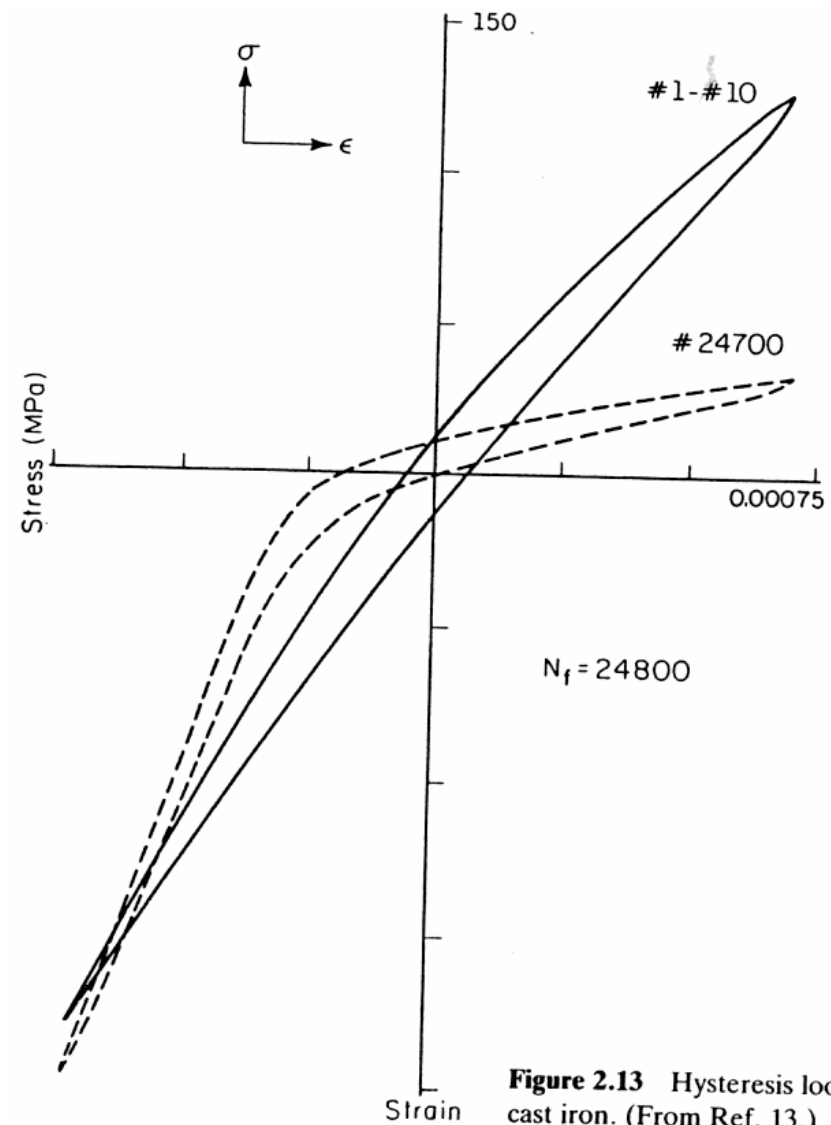
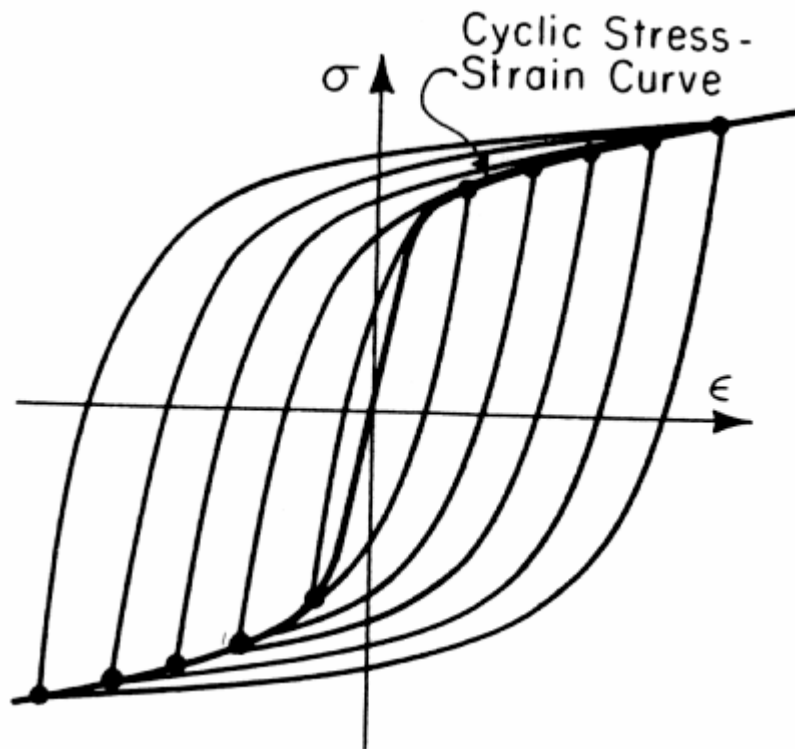
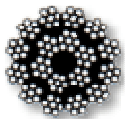


Figure 2.13 Hysteresis loops of gray cast iron. (From Ref. 13.)



Úvod

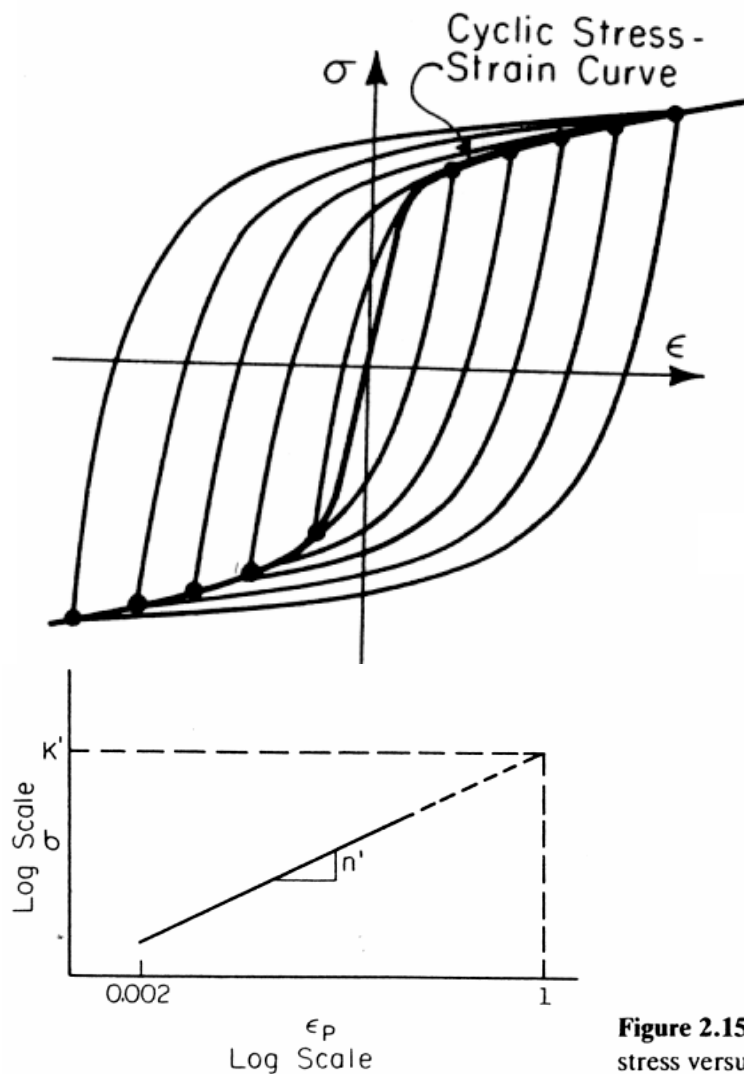


Figure 2.15 Log-log plot of true cyclic stress versus true cyclic plastic strain.

$$\sigma = K'(\epsilon_p)^{n'}$$

$$\epsilon_p = \left(\frac{\sigma}{K'}\right)^{1/n'}$$

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

σ = cyclically stable stress amplitude

ϵ_p = cyclically stable plastic strain amplitude

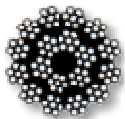
K' = cyclic strength coefficient

n' = cyclic strain hardening exponent

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

n ... exponent cyklického zpevnění

K ... koeficient cyklického zpevnění



Úvod

Basquin, 1910:

$$\frac{\Delta\sigma}{2} = \sigma'_f(2N_f)^b$$

$\frac{\Delta\sigma}{2}$ = true stress amplitude

$2N_f$ = reversals to failure (1 rev = $\frac{1}{2}$ cycle)

σ'_f = fatigue strength coefficient

b = fatigue strength exponent (Basquin's exponent)

Coffin a Manson, po r. 1950:

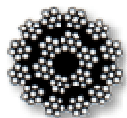
$$\frac{\Delta\epsilon_p}{2} = \epsilon'_f(2N_f)^c$$

$\frac{\Delta\epsilon_p}{2}$ = plastic strain amplitude

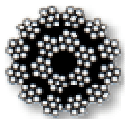
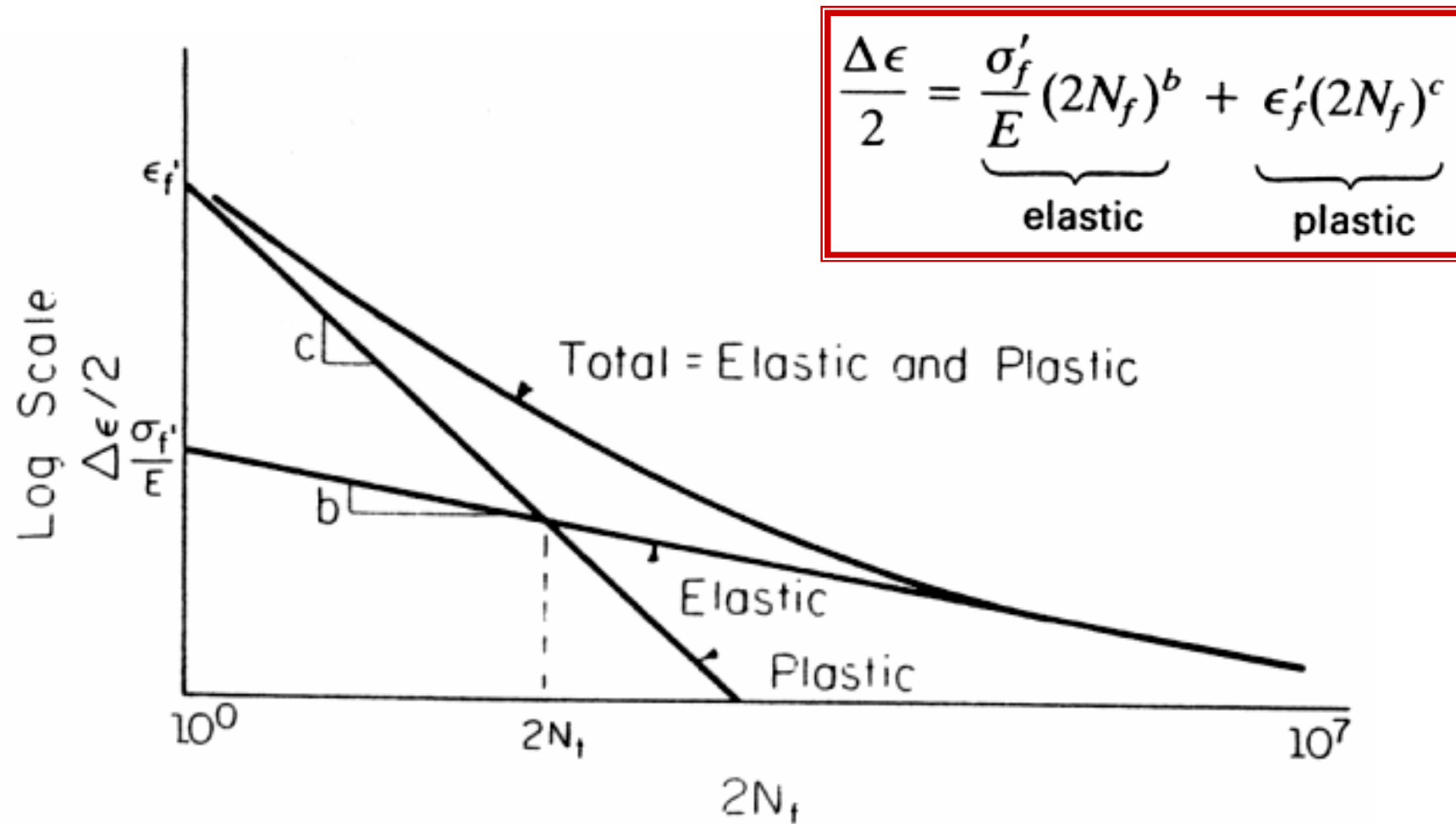
$2N_f$ = reversals to failure

ϵ'_f = fatigue ductility coefficient

c = fatigue ductility exponent



Úvod



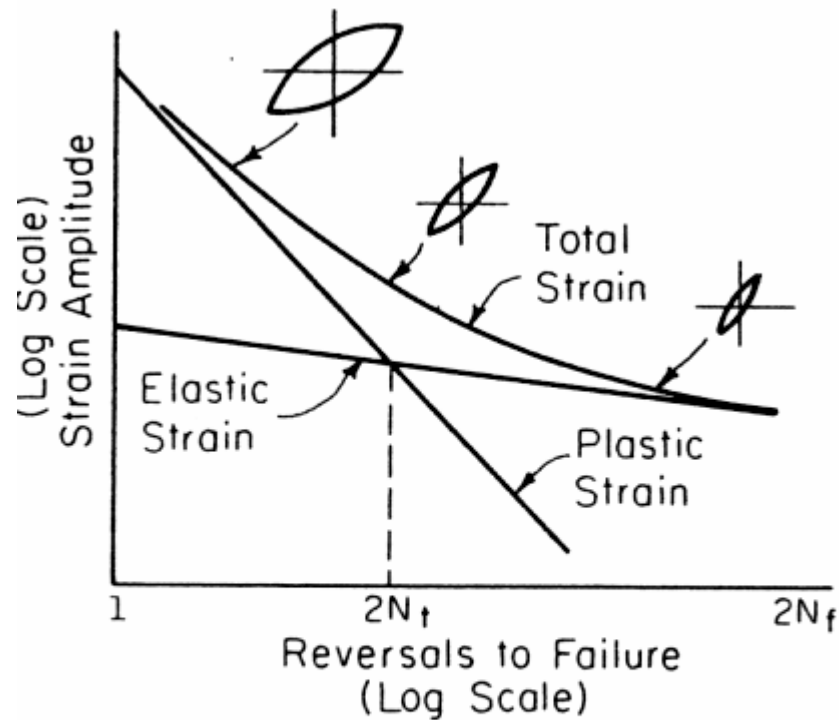


Figure 2.18 Shape of the hysteresis curve in relation to the strain-life curve. (From Ref. 17.)

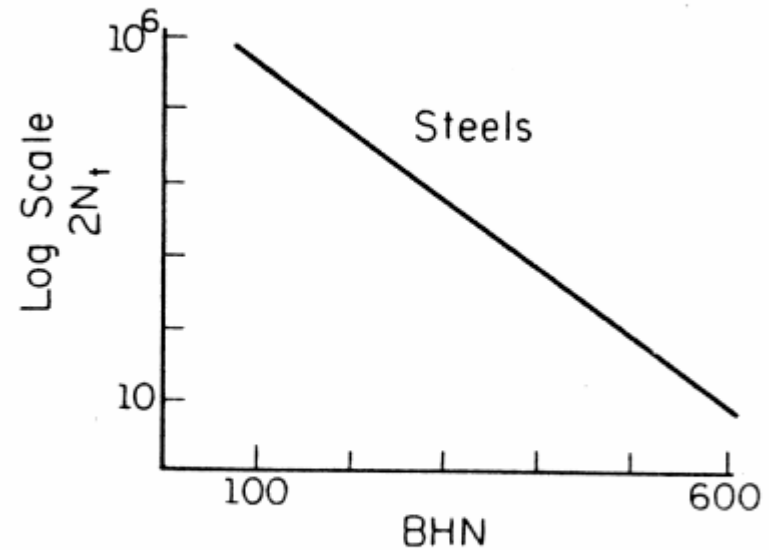
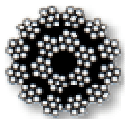
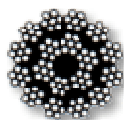
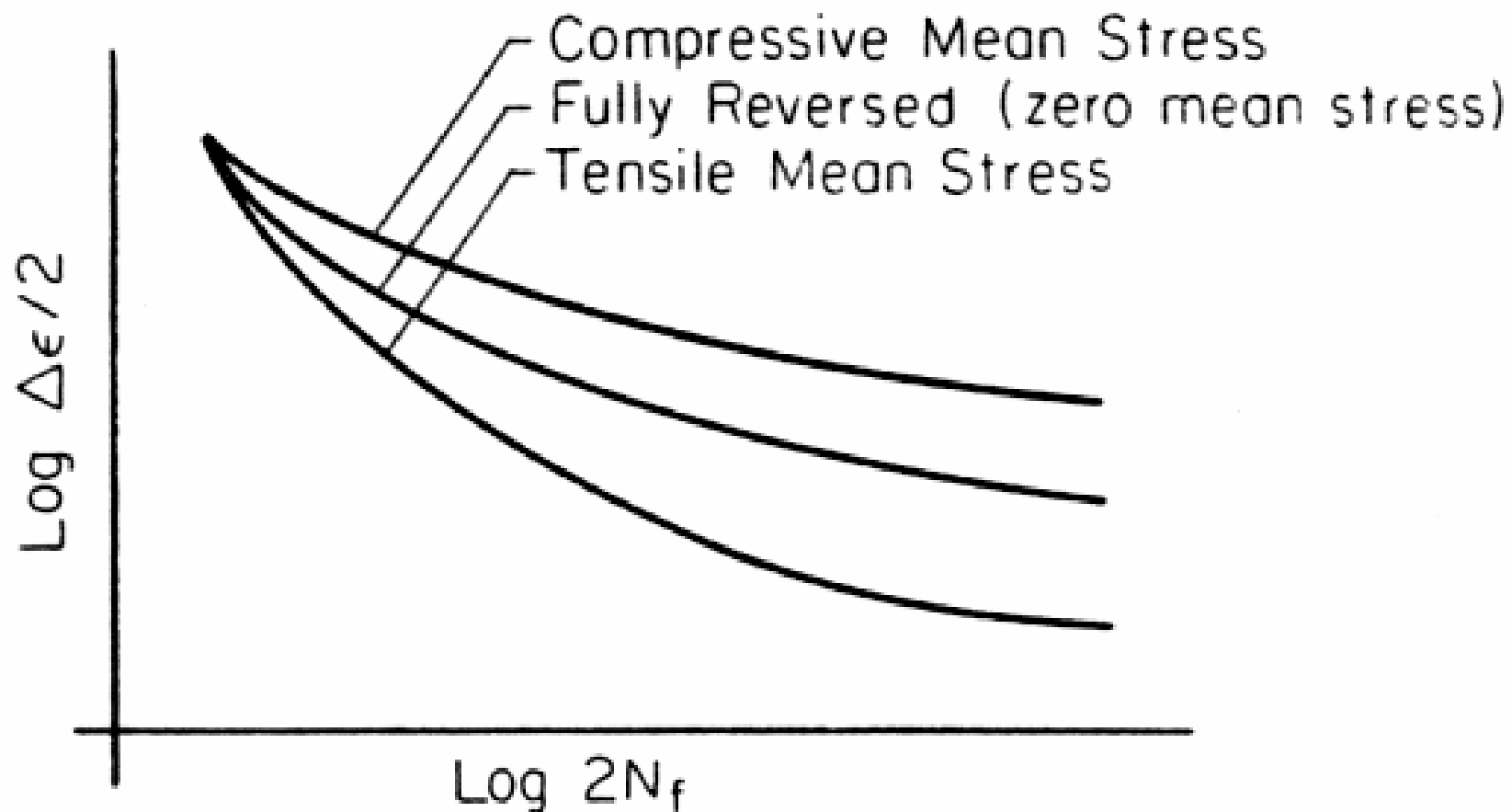


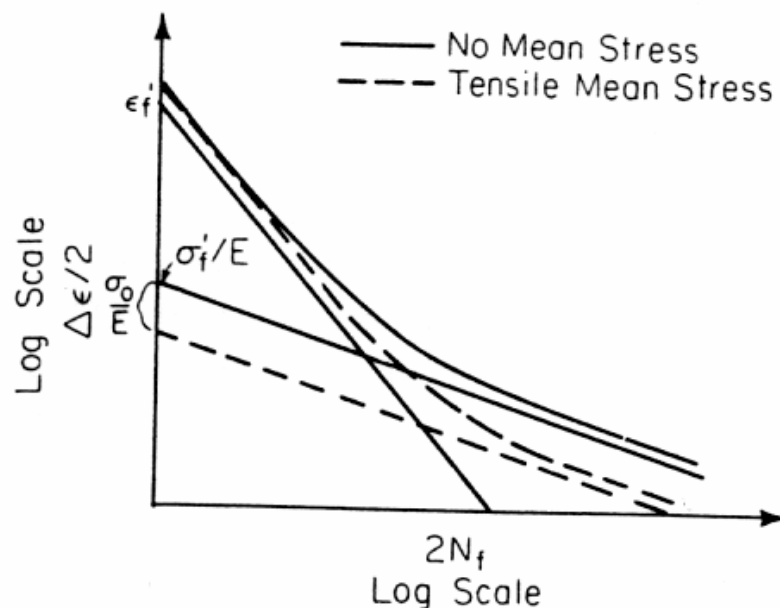
Figure 2.19 Relationship between transition life and hardness for steels. (From Ref. 17.)



Vliv středního napětí



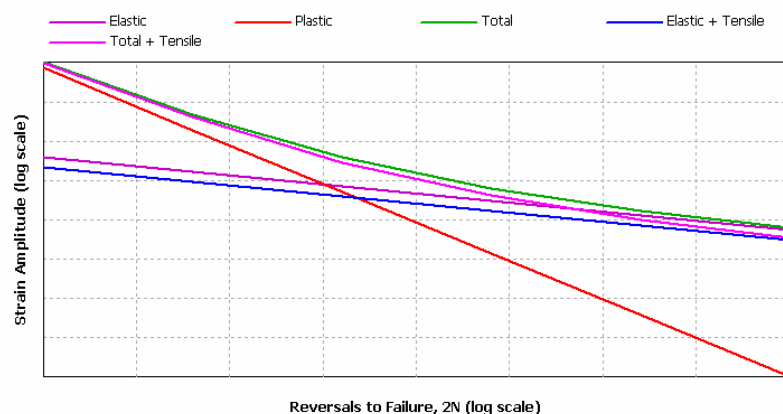
Vliv středního napětí



Morrow:

$$\epsilon_a = \frac{\sigma'_f}{E} \left(1 - \frac{\sigma_m}{\sigma'_f} \right) (N_f)^b + \epsilon'_f (2N_f)^c$$

Figure 2.23 Morrow's mean stress correction to the strain-life curve for a tensile mean.



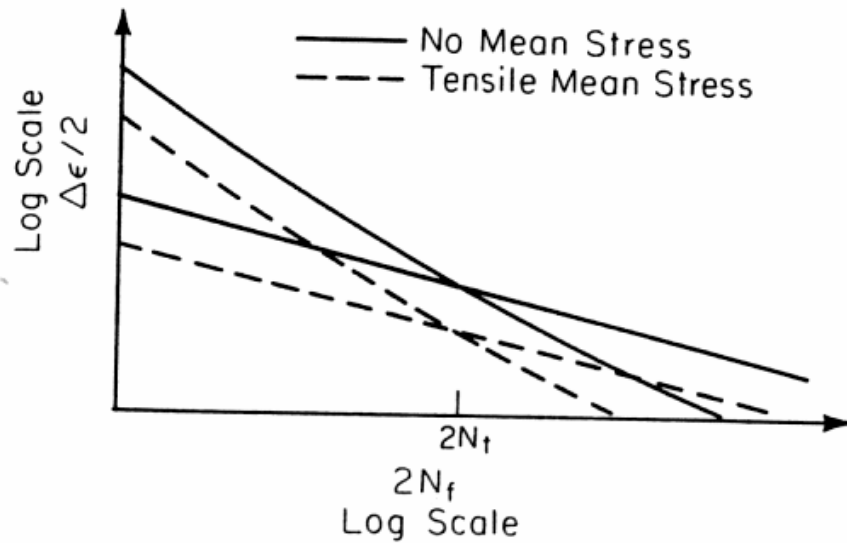
$$\frac{\Delta \epsilon}{2} = \underbrace{\frac{\sigma'_f}{E} (2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon'_f (2N_f)^c}_{\text{plastic}}$$

$$\frac{\Delta \sigma}{2} = \sigma'_f (2N_f)^b \quad \frac{\Delta \epsilon_e}{2} = \frac{\Delta \sigma}{2E} = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b$$

Implementace v Ansysu



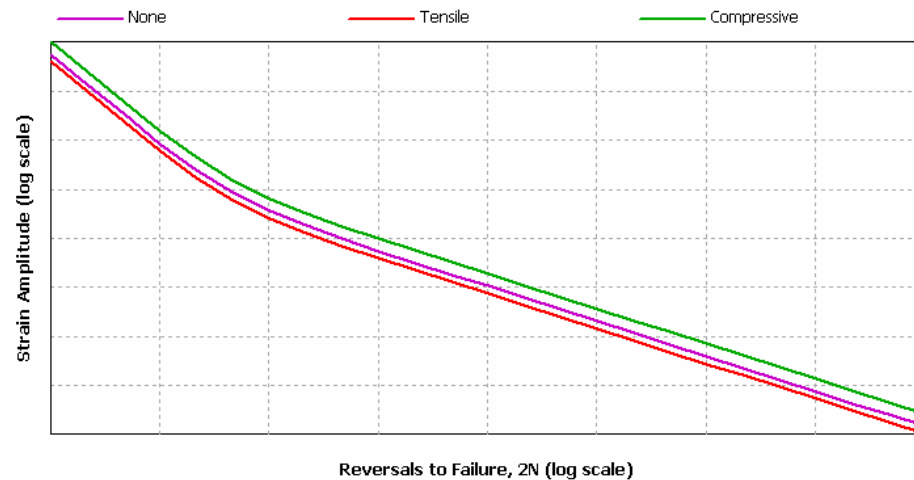
Vliv středního napětí



Smith, Watson a Topper (SWT):

$$\sigma_{\max} \epsilon_a = \frac{(\sigma'_f)^2}{E} (N_f)^{2b} + \sigma'_f \epsilon'_f (2N_f)^{b+c}$$

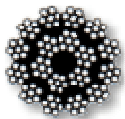
Figure 2.25 Mean stress correction for independence of elastic/plastic strain ratio from mean stress.



$$\frac{\Delta \epsilon}{2} = \underbrace{\frac{\sigma'_f}{E} (2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon'_f (2N_f)^c}_{\text{plastic}}$$

$$\sigma_{\max} = \frac{\Delta \sigma}{2} = \sigma'_f (2N_f)^b$$

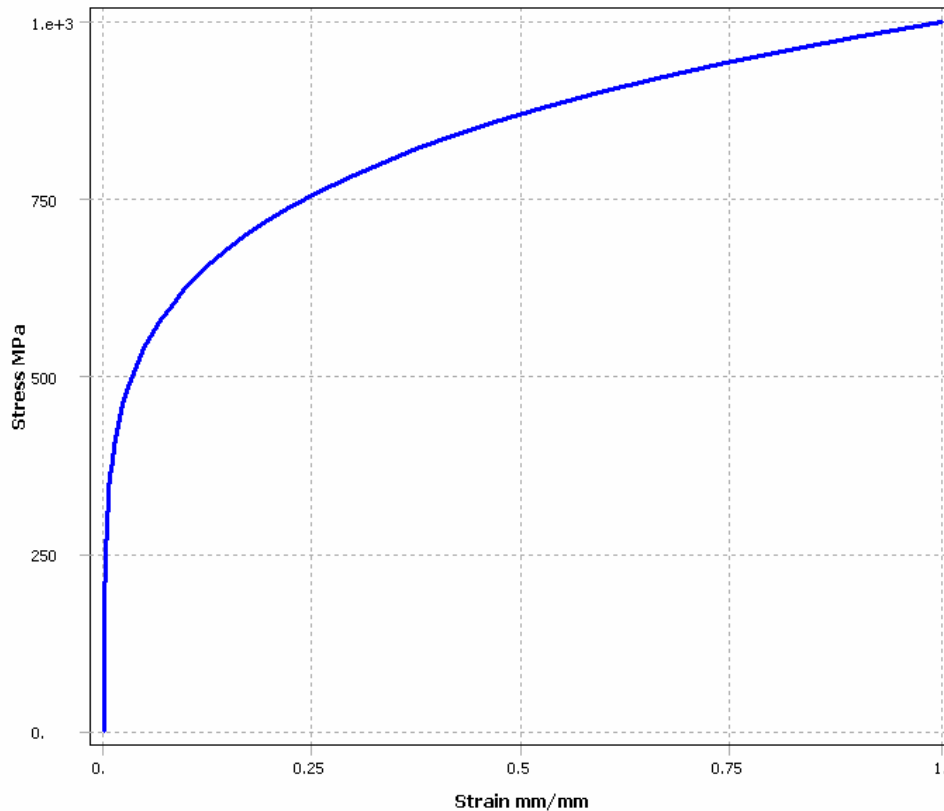
Implementace v Ansysu



Poznámka

Plastizace materiálu není zahrnuta ve statickém výpočtu. Pro nízkocyklové výpočty je nutné vložit do softwaru (v našem případě Ansys) cyklickou křivku napětí-deformace. Tu je možné v nejjednodušším případě vložit jako **bilineární** či **multilineární**.

Jinou formou náhrady skutečné materiálové křivky může být **Ramberg – Osgoodův** vztah:



$$\varepsilon_a = \frac{\Delta \varepsilon}{2} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'} \right)^{\frac{1}{n'}}$$

n' ... exponent cyklického zpevnění

K' ... koeficient cyklického zpevnění



Použitá literatura

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